Performance Based Learning and Assessment Task

How Tall Is It?

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:

Working in pairs, students will make and use a "Trig-o-no-Meter" (clinometer) to closely approximate the height of tall objects without leaving the ground. This activity will demonstrate the practical usefulness of right triangle trigonometry in real-world problems and provide practice in right triangle trigonometry calculations.

II. UNIT AUTHOR:

Jerry Dawson – Glen Allen High School – Henrico County Public Schools

III. COURSE:

Geometry

IV. CONTENT STRAND:

G.8 The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

V. OBJECTIVES:

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Solve problems involving right triangles, using sine, cosine, and tangent ratios.

VI. REFERENCE/RESOURCE MATERIALS:

Protractor, string, and hex nut or washer (for weighting the string)
10'- long rope
Clipboard & pencil
Data Collection handout (see below)
Calculator with trig functions

VII. PRIMARY ASSESSMENT STRATEGIES:

Task checklist, or rubric, (attached) will be used to assess:
1) the student pair's construction of a "Trig-o-no-meter" using written directions and provided materials
2) student accuracy in making angle of elevation measurements on a collection of tall objects in and around the school.
3) student accuracy of their calculation results as compared to other student findings
4) student understanding of the principles behind the Trig-o-no-meter as communicated in response to questions posed throughout the task.

VIII. EVALUATION CRITERIA:

See VII above, and Assessment Checklist (below).

IX. INSTRUCTIONAL TIME:

One 90-minute class period.

How Tall Is It?

Strand
Geometry

Mathematical Objective(s)
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Solve problems involving right triangles, using sine, cosine, and tangent ratios.

Related SOL
G.8 The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

NCTM Standards
- Use trigonometric relationships to determine lengths and angle measures.
- Analyze precision, accuracy, and approximate error in measurement situations.
- Build new mathematical knowledge through problem solving.

Additional Objectives for Student Learning (include if relevant; may not be math-related):
None

Materials/Resources
Protractors, string, and hex nuts or heavy washers (for weighting the string)
Rulers, yardstick, and meter stick
Precut lengths of rope (10', 20', and 30' – three each)
Clipboards & pencils
Data Collection handout (see below)
Classroom set of calculators (with trig functions)
Classroom document camera and smart board
School building

Assumption of Prior Knowledge
- Measuring lengths with ruler, yardstick, and/or meter stick.
- Use of a protractor to determine the measure of angles.
- The definition of sine, cosine, and tangent trigonometric functions.
- The use of sine, cosine, tangent in solving right triangle trigonometry word problems.

Introduction: Setting Up the Mathematical Task

- "Today we are going to apply right triangle trigonometry to complete a typical task faced by someone in their job. That job could be: Architect, Draftsman, Artist, Construction Manager, Cost estimator, etc. Grab a notebook and pencil, and let's take a walk..."
- Take the class to the front lobby of the school building, stopping at the LEED plaque on the wall. "You may or may not know, but Glen Allen High School was designed and built to LEED standards, and received LEED certification upon it's opening in 2010.

  "LEED, or Leadership in Energy & Environmental Design, is a green building certification program that recognizes best-in-class building strategies and practices." ~ U. S. Green Building Council
- "Henrico County is very proud of this accomplishment, as it is the first school building in Central Virginia with this distinction. However, we have a problem. In a recent LEED audit, they discovered that the wrong windows were installed on the front of the school building. Instead of high-efficiency windows, mid-efficiency windows were installed by mistake. Obviously they must be replaced. The building contractors are willing to change out the windows, but they need to know the rough sizes of the openings in order to schedule their manufacture. Detailed measurements will be made at a later date, but they are waiting for a quick answer. As our summer intern, you need to provide the answer. As is typical, your supervisor expects that you figure out how to accomplish this task accurately and quickly, without having to be told what to do."

Student Exploration

Small Group Work (teams of three students)

- "You and your two partner interns just drove up from the company office. It is summer, and the building is locked. Spread out, walk around, come up with a plan of attack, sketch some ideas on paper, and in 5 minutes we will head back to the office where you will report in to your supervisor."
- As they work, stroll from team to team and observe the dialogue. Suggest that each group makes a sketch of the building, to help explain things to their supervisor back at the office. After five minutes, take the class back inside to the "office" (classroom).
- Pretending to be the supervisor, welcome the interns and say, "Welcome back. I was on my way down the hall. Give me five minutes, then I want to hear what you found at the school site, and how you are going to complete the task. Do you have any quick questions before I go?" (Answer team questions. Expect questions about tools available for the job. In the classroom, typical "tools" are casually set out on a side table (rulers, protractors, rope and string, heavy washers, calculators, etc. Expect that your answers will force them to rethink their approach to the task.)
- Again, as they work, stroll around the classroom and observe the dialogue.

Whole Class Sharing/Discussion

- "Sorry for the delay. Alright, what did you find, what's your plan, and what do you need?" Call on a random team to report, starting the class dialogue. Use the classroom document camera to display sketches for conversation sake.
- Teacher carefully discounts solutions requiring direct measurement of the height of the building features ("no ladder," "no scissor lift," etc.) and instead looks for solutions involving right triangle trigonometry, because we can measure along the ground. Teacher involves as many teams as possible in contributing toward an eventual strategy/solution. Teacher helps discover available tools that might assist in the endeavor.
- Key Questions:
  - "If we want to use right triangle trigonometry, what information will we need?" *(angle of elevation, and distance to the object measured)*
  - "Why do we need these?" *(since we are solving for a length, we need an angle and another length to use trig)*
  - "What trig function will be most helpful?" *(TOA tangent, since we have Adjacent, and we are solving for Opposite)*
  - "How can we obtain the angle of elevation?" or "what do we use to measure angles?" *(protractor)*
- Teacher says, "We have protractors." Hand them out. "So how will this tell me the angle of elevation?" Give teams some time to think about how this could work. Circulate around the room and observe the dialogue.
- Ask, "Alright, how is this protractor going to help us?" Guide class to discover the elements of a "trig-ono-meter" (clinometer) to measure the angle of elevation.

Small Group Work

- "You and your partner must now build a "trig-ono-meter" to measure the elevations on the front of the school building. You have string and heavy washers, and five minutes."
- After five minutes, "Now, how are we going to measure the ground distance to the building using our available materials? Think for a moment, then let's hear from each team."
- Collect ideas from each team on measuring the ground distance. Possible solutions:
  - Yardstick. Since there is one yardstick in the office, one team may use it.
  - Meter stick. Since there is one meter stick in the office, one team may use it.
  - Tape measure. Since there is one tape measure in the office, one team may use it.
  - Walk off in "feet." Allow one team to use this method. (Use ruler to measure someone's actual foot)

    - Rope. Remaining teams can use measured length of rope. Have lengths of 10, 20, and 30 feet available.
- Before heading back to the site, please test you method here in the office. “Measure” the height of the ceiling in this room using your intended procedure and tools. Show me all of your calculations. If you are not accurate, you will need to repeat the task until you get it right.
- When all are ready, take the class back out front and have them record their measurements on their sketch of the building façade, using the Data Collection Sheet (below). As teacher circulates, ask “what potential sources of error might affect your results?” Allow 20 minutes. Students who finish sooner than others should begin their trig calculations and sanity check their results.
- Back in the “office,” team should complete their calculations. The teacher circulates, giving assistance when needed, and asking probing questions to test understanding.

Closing:

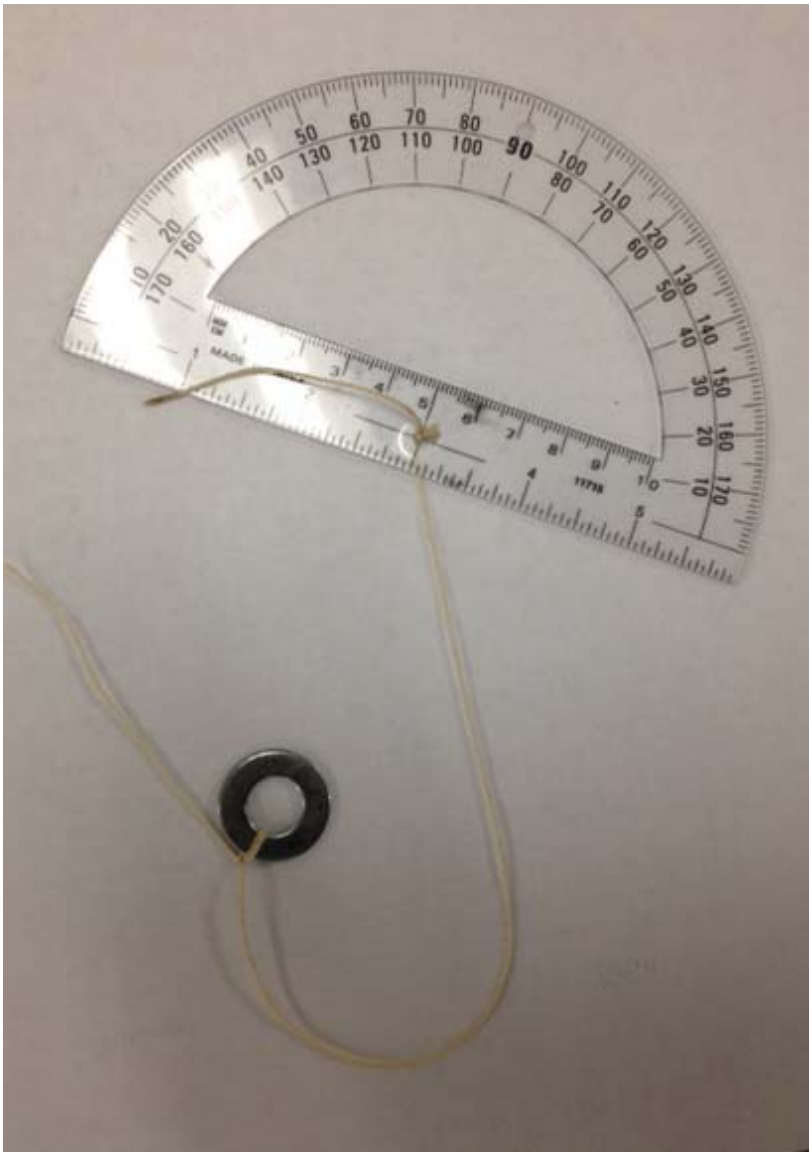
- When all teams are complete, each team reports on their results for each of the required heights. Teacher records results in a spreadsheet on the smart board.
- The class then evaluates the accuracy of the results. Deviations are discussed, along with methods to correct for potential errors in measurement methods. The class then tries to determine which of the various methods of measurement was the most accurate.
- Students submit their work for the day.
- In their thinking journals, students address the following writing prompts:
    - I was most helpful to my team today by…
    - I still have questions about…
    - How are you a better geometer today than yesterday?
- In closing, the teacher summarizes the purposes and outcomes of the activity, highlighting the real-world application of right triangle trigonometry to solve problems and obtain hard-to-get measurements easily using simple tools.

Assessment List and Benchmarks

- Assessment Checklist (see below)
- Hypothetical Student Example (see below)

Hypothetical Student Example – Trig-ono-Meter

Completed Trig-ono-Meter:



Trig-ono-Meter

Data Collection Sheet

Team Members: _____

Calculations:

Use back if needed

Hypothetical Student Example – Data Collection Sheet

Trig-ono-Meter

Data Collection Sheet

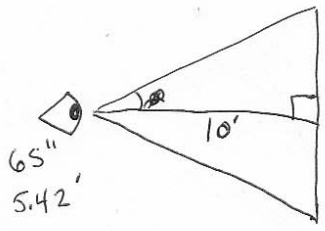
Team Members: SAMPLE

		30.78	72° ~~78°~~
57°	15.40		57°
56°	14.83		56°
51½°	12.57	SAME	51½
48½°	11.30		48½
43°	9.33		43°
		X	HIDDEN BY OVERHANG
18°	3.25	3.64	20°
16°	2.87	1.76	10°
		1.32	7½°
		0.70	4°
-9°	-1.58		
-17°	-3.06	-3.25	-18°
-27°	-5.10	-4.67	-25°

AT : 10'

Calculations:

Use back if needed



$\tan\theta = \frac{x}{10}$

Assessment Checklist

Team Members: _____

Student Exploration

Task	Points Earned		
Team develops and presents approach for completing the measuring task to supervisor.	0 No serious approach presented	5 Approach uses impractical direct measurement	10 Approach feasible using indirect measurement
Team actively contributes to the development of a method for obtaining the height measurements given the constraints that exist.	0 Team does not contribute	5 Team somewhat involved in large-group process	10 Team constructively contributes to large-group process

Trig-ono-Meter

Task	Points Earned		
Team completes construction of a functioning trig-ono-meter based on class discussion and materials provided.	0 No Trig-ono-Meter built	5 Non-functioning Trig-ono-Meter	10 Working Trig-ono-Meter
Team tests trig-ono-meter and method against a known height sample (classroom ceiling).	0 No test completed	3 Test results >6" from actual	5 Test results within 6" of actual

Data Collection

Task	Points Earned		
Team completes measurement of all required angle of elevation using Data Collection Sheet.	0 Team obtains no field measurements	10 Team obtains about half of required field measurements	20 Team obtains all necessary field measurements

Height Calculations

Task	Points Earned		
Team completes calculation of all required heights using accurate right triangle trigonometry ratios.	0 No calculations completed	15 Some necessary calculation completed	25 All necessary calculation completed
Team recognizes potential measurement errors, adjusts calculations as necessary, and determines the correct heights within six inches.	0 No errors recognized or mitigated	10 Some potential errors recognized and mitigated	20 All potential errors recognized and mitigated

	Points Possible	Points Earned
Team Total	100	